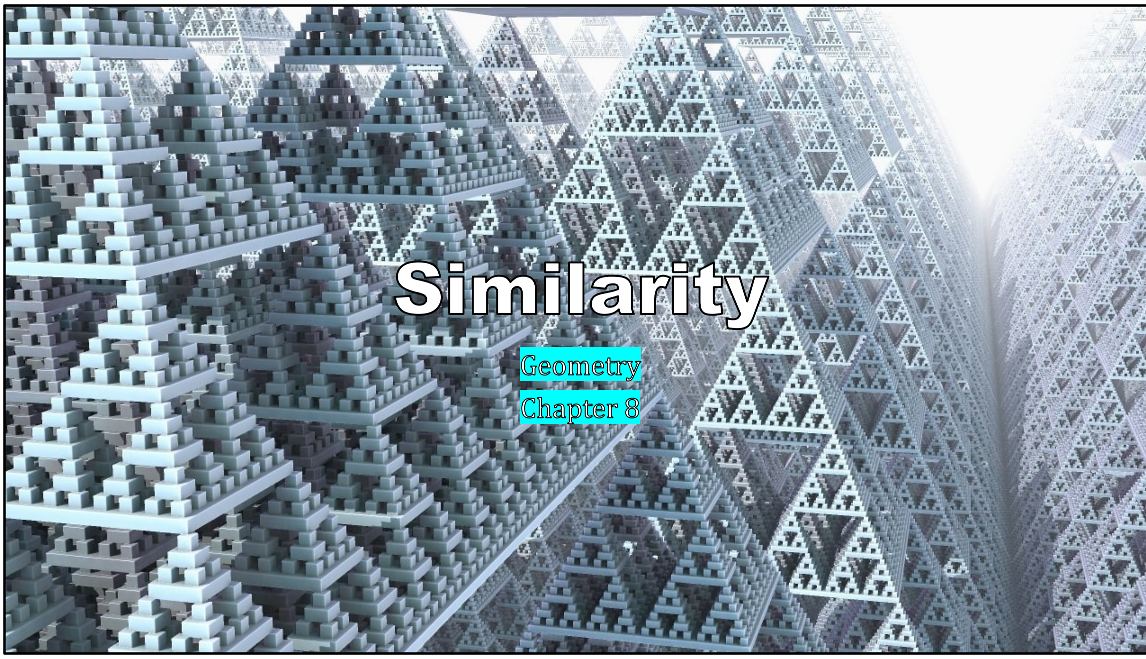
Similarity
Geometry
Chapter 8

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Geometry*
  - *By Larson and Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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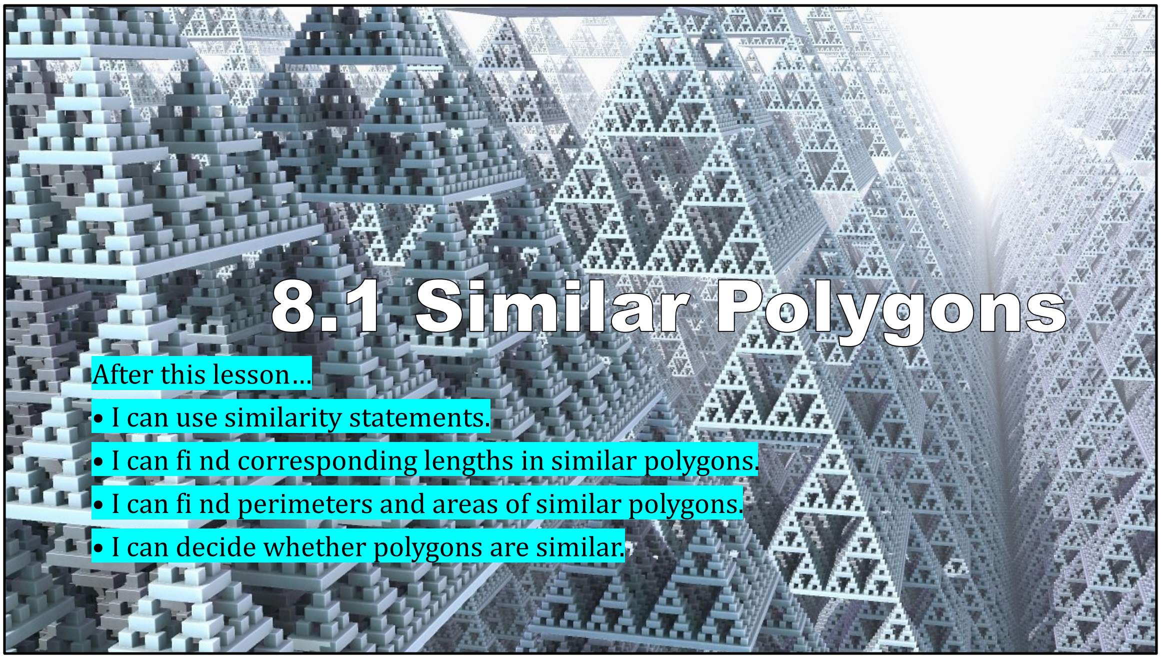


8.1 Similar Polygons

After this lesson…

- I can use similarity statements.
- I can find corresponding lengths in similar polygons.
- I can find perimeters and areas of similar polygons.
- I can decide whether polygons are similar.

8.1 Similar Polygons

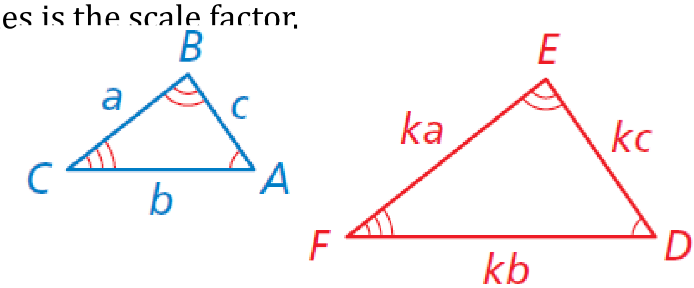
- When I show the same thing on the overhead projector and the computer monitor, the projected image is larger than what is on the screen. The image is of a different size, but the same shape as what I write. They are similar.

Have the overhead ready

8.1 Similar Polygons

- Similar figures
- When two figures are the same shape but different sizes, they are similar.

- Similar polygons (~)
- Polygons are similar iff corresponding angles are congruent and corresponding sides are proportional.
- Ratio of lengths of corresponding sides is the scale factor.

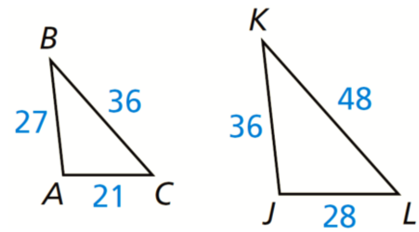


- Angles
  - $\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$
- Ratios of side lengths (scale factor)
  - $\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = k$

8.1 Similar Polygons

$\triangle ABC \sim \triangle JKL$

a. Find the scale factor from $\triangle ABC$ to $\triangle JKL$.



b. List all pairs of congruent angles.

c. Write the ratios of the corresponding side lengths in a statement of proportionality.

Try #2

$$\frac{JK}{AB} = \frac{36}{27} = \frac{4}{3}$$

$$\angle A \cong \angle J, \angle B \cong \angle K, \angle C \cong \angle L$$

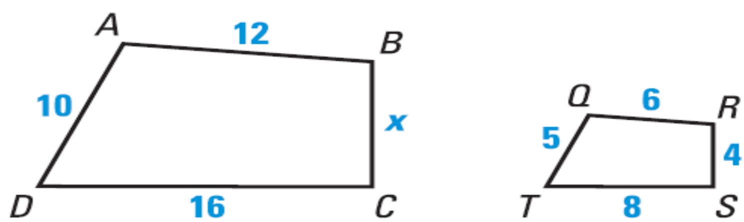
$$\frac{JK}{AB} = \frac{KL}{BC} = \frac{LJ}{CA}$$

8.1 Similar Polygons

- $ABCD \sim QRST$
- What is the scale factor of $QRST$ to $ABCD$?

- Find x.

- Try #4

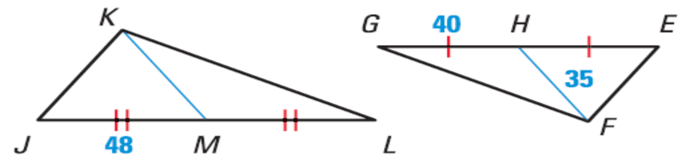


Scale factor: $\frac{12}{6} = \frac{2}{1}$

x: $\frac{2}{1} = \frac{x}{4}$ → x = 8

8.1 Similar Polygons

- $\Delta JKL \sim \Delta EFG$. Find the length of the median $\overline{KM}$.



- Try #8

$$\frac{48}{40} = \frac{KM}{35}$$
$$40 \cdot KM = 48 \cdot 35$$
$$40 \cdot KM = 1680$$
$$KM = 42$$

8.1 Similar Polygons

Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

- If $\Delta ABC \sim \Delta DEF$, then $\frac{DE}{AB} = \frac{\text{Perimeter } \Delta DEF}{\text{Perimeter } \Delta ABC}$

Area of Similar Polygons

If two polygons are similar, then the ratio of their areas is equal to the squares of the ratios of their corresponding side lengths.

- If $\Delta ABC \sim \Delta DEF$, then $\left(\frac{DE}{AB}\right)^2 = \frac{\text{Area } \Delta DEF}{\text{Area } \Delta ABC}$

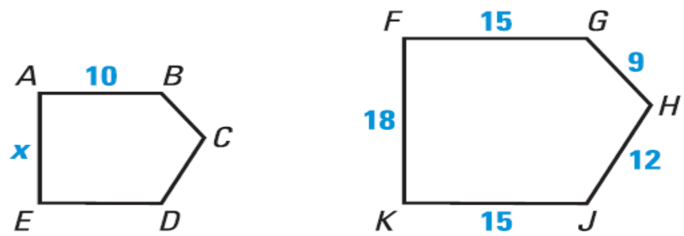
8.1 Similar Polygons

$ABCDE \sim FGHJK$, the area of $FGHJK$ is 318 in^2

- Find the scale factor of $FGHJK$ to $ABCDE$

- Find the perimeter of $ABCDE$

- Find the area of $ABCDE$



- Try #18

Scale factor: $\frac{10}{15} = \frac{2}{3}$

Perimeter: $\frac{2}{3} = \frac{P}{15+9+12+15+18} \rightarrow \frac{2}{3} = \frac{P}{69} \rightarrow 3P = 138 \rightarrow$ P = 46 in^2

Area: $\left(\frac{2}{3}\right)^2 = \frac{A}{318} \rightarrow \frac{4}{9} = \frac{A}{318} \rightarrow 9A = 1272 \rightarrow A = 141.3\ in^2$

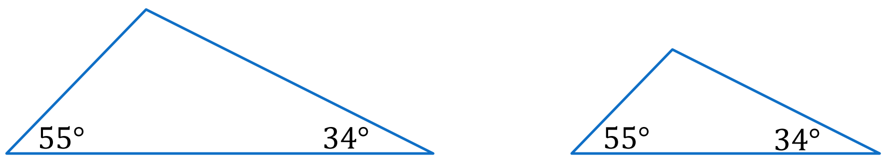
8.2 Proving Triangle Similarity by AA

After this lesson…

- I can use similarity transformations to prove the Angle-Angle Similarity Theorem.
- I can use angle measures of triangles to determine whether triangles are similar.
- I can solve real-life problems using similar triangles.

8.2 Proving Triangle Similarity by AA

- Draw two triangles with two pairs of congruent angles. Measure the corresponding sides. Are they proportional? Are the triangles similar?

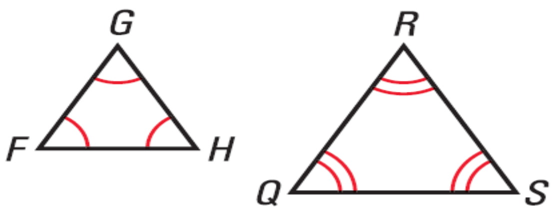


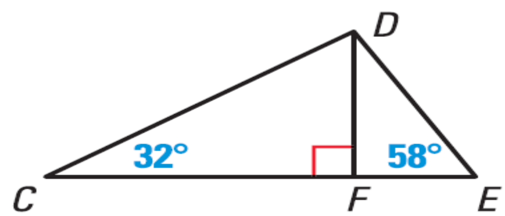
AA Similarity

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

8.2 Proving Triangle Similarity by AA

- Show that the triangles are similar. Write a similarity statement.





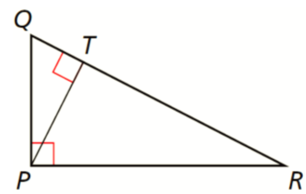
- Try #2

ΔFGH ~ ΔQRS by AA Similarity

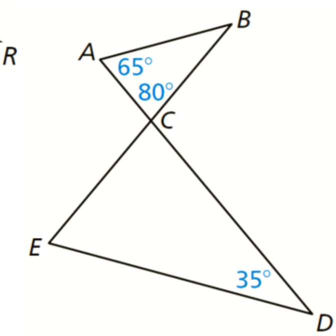
m∠CDF = 58 by Triangle Sum Theorem
ΔCDF ~ ΔDEF by AA Similarity

8.2 Proving Triangle Similarity by AA

- Show that the triangles are similar. Write a similarity statement.
- $\triangle QPR$ and $\triangle QTP$



- $\triangle ABC$ and $\triangle EDC$



- Try #6

$\angle Q \cong \angle Q$ by the Reflexive Property of Angle Congruence. $\angle QPR \cong \angle QTP$ by the Right Angles Congruence Theorem. So, $\triangle QPR \sim \triangle QTP$ by the AA Similarity Theorem.

$\angle ACB \cong \angle ECD$ by the Vertical Angles Congruence Theorem. $m\angle B = 35°$ by the Triangle Sum Theorem. Because $m\angle B$ and $m\angle D$ both equal 35°, $\angle B \cong \angle D$. So, $\triangle ABC \sim \triangle EDC$ by the AA Similarity Theorem.

8.2 Proving Triangle Similarity by AA

- You can use similar triangles to find things like the height of a tree by using shadows. You put a stick perpendicular to the ground. Measure the stick and the shadow. Then measure the shadow of the tree. The triangles formed by the stick and the shadow and the tree and its shadow are similar so the height of the tree can be found by ratios. Suppose we use a meter stick. The stick's shadow is 3 m. The tree's shadow is 150 m. How high is the tree?

- Try #20

$$\frac{\text{Tree Shadow}}{\text{Stick Shadow}} = \frac{\text{Tree Height}}{\text{Stick Height}}$$

$$\frac{150}{3} = \frac{x}{1}$$

$$x = 50\ m$$

8.3 Proving Triangle Similarity by SSS and SAS

After this lesson…

• I can use the SSS and SAS Similarity Theorems to determine whether triangles are similar.

8.3 Proving Triangle Similarity by SSS and SAS

SSS Similarity

If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.

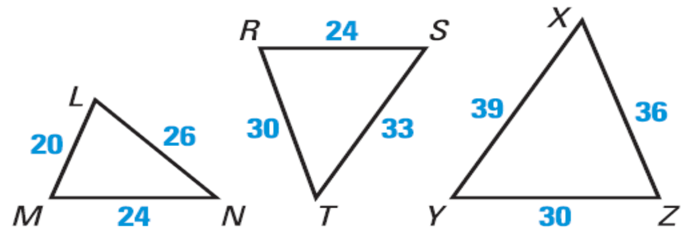
SAS Similarity

If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

SSS Similarity - That's what happens when you enlarge a drawing.

8.3 Proving Triangle Similarity by SSS and SAS

- Which of the three triangles are similar?



- Try #2

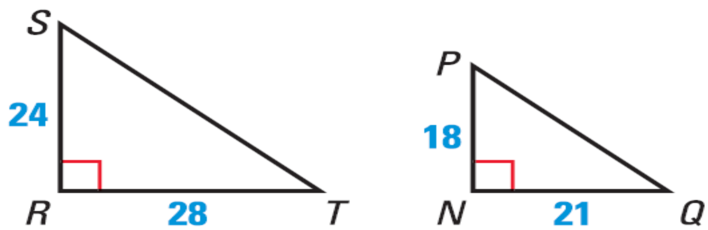
Try ΔLMN and ΔRST: $\frac{20}{24} = \frac{24}{30} = \frac{26}{33}$ This is not true.

Try ΔLMN and ΔXYZ: $\frac{20}{30} = \frac{24}{36} = \frac{26}{39}$ This is true. ΔLMN ~ ΔYZX

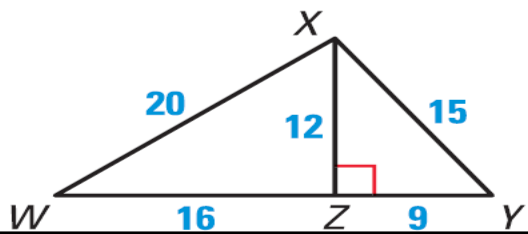
Try ΔXYZ and ΔRST: $\frac{30}{24} = \frac{36}{30} = \frac{39}{33}$ This is not true.

8.3 Proving Triangle Similarity by SSS and SAS

- Explain how to show that the indicated triangles are similar.
- $\Delta SRT \sim \Delta PNQ$



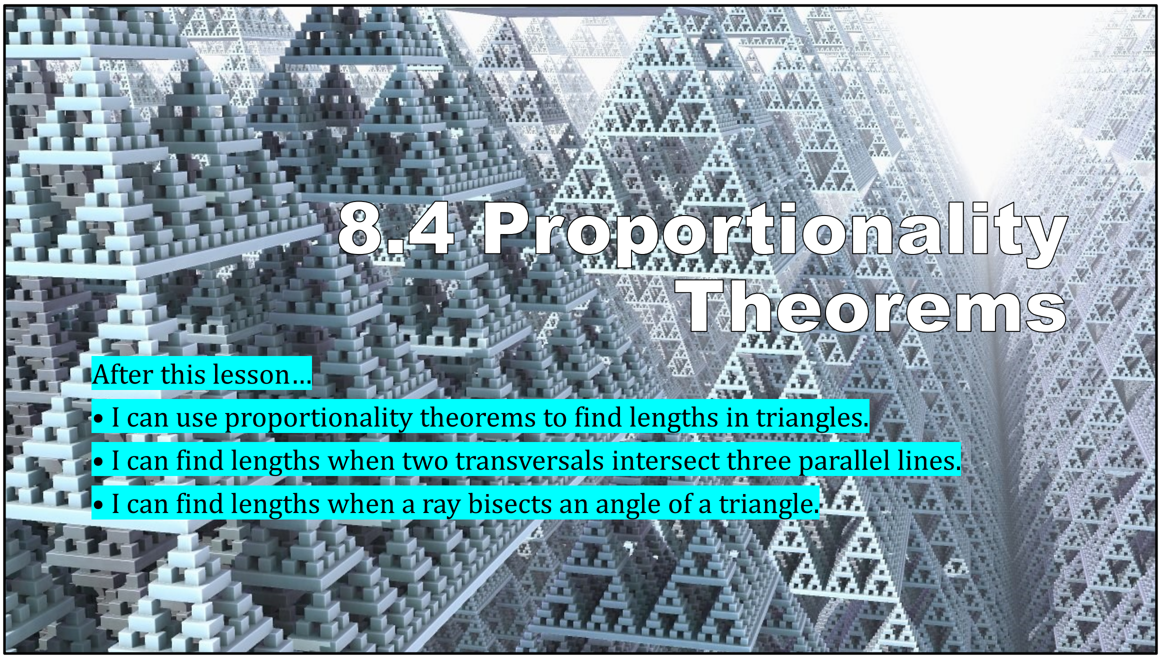
- $\Delta XZW \sim \Delta YZX$



- Try #9

$\frac{RS}{NP} = \frac{24}{18} = \frac{4}{3}; \frac{RT}{NQ} = \frac{28}{21} = \frac{4}{3}; \angle R \cong \angle N$; SAS Similarity

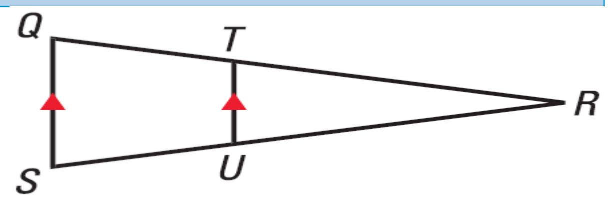
$\frac{XZ}{YZ} = \frac{12}{9} = \frac{4}{3}; \frac{ZW}{ZX} = \frac{16}{12} = \frac{4}{3}; \frac{XW}{YX} = \frac{20}{15} = \frac{4}{3}$; SSS Similarity (or SAS Similarity)


8.4 Proportionality Theorems
After this lesson…
• I can use proportionality theorems to find lengths in triangles.
• I can find lengths when two transversals intersect three parallel lines.
• I can find lengths when a ray bisects an angle of a triangle.

8.4 Proportionality Theorems

Triangle Proportionality Theorem

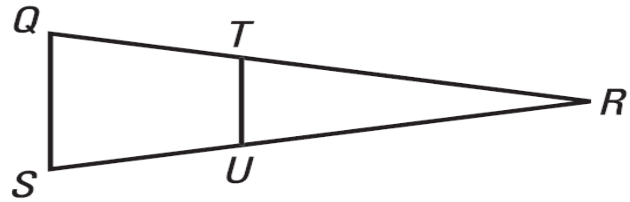
If a line is parallel to a side of a triangle, then it separates the other two sides into proportional segments.



- And the converse is also true. Proportional segments → line parallel to the third side.

8.4 Proportionality Theorems

- In ΔRSQ with chord TU, $QR = 10$, $QT = 2$, $UR = 6$, and $SR = 12$. Determine if $\overline{QS} \parallel \overline{TU}$.



- Try #4

$$TR = 10 - 2 = 8, US = 12 - 6 = 6$$

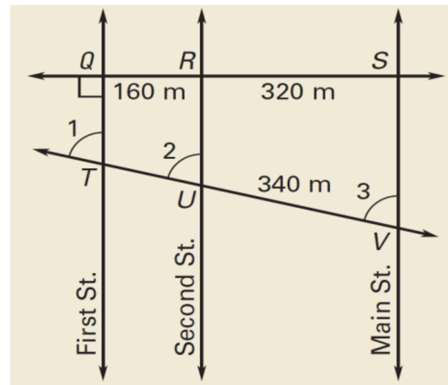
$$\frac{TR}{QT} = \frac{RU}{US} \rightarrow \frac{8}{2} = \frac{6}{6} \rightarrow 4 = 1$$

False, not parallel

8.4 Proportionality Theorems

If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

- Using the information in the diagram, find the distance *TV*.



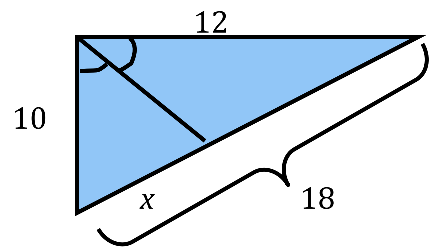
- Try #12

$$\frac{TU}{340} = \frac{160}{320}$$
$$320TU = 54400$$
$$TU = 170$$
$$TV = TU + UV = 170 + 340 = 510\,m$$

8.4 Proportionality Theorems

An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides.

- Find x



- Try #18

$$\frac{10}{x} = \frac{12}{18 - x} \rightarrow 10(18 - x) = 12x \rightarrow 180 - 10x = 12x \rightarrow 180 = 22x \rightarrow x = \frac{180}{22} = 8.18$$